Initial and Final Value Theorem -

(a) Initial Value Theorem :-

If $L\{f(t)\} = F(s)$.

then $\lim_{t \to 0} f(t) = \lim_{s \to \infty} [sF(s)]$ Provided the limit exist

Proof: As we known that -

$$L\{f'(t)\} = sL\{f(t)\} - f(0)$$

$$\int_0^\infty e^{-st} f'(t)\,dt = sF(s) - f(0)$$

$$\lim_{s \to \infty} \int_0^\infty e^{-st} \cdot f'(t)\,dt = \lim_{s \to \infty} [sF(s) - f(0)]$$

or $$\int_0^\infty \left(\lim_{s \to \infty} e^{-st}\right) f'(t)\,dt = \lim_{s \to \infty} [sF(s)] - f(0)$$

$$0 = \lim_{s \to \infty} [sF(s)] - f(0)$$

or, $$\lim_{s \to \infty} [sF(s)] = f(0) = \lim_{t \to 0} f(t).$$

$$\Rightarrow \boxed{\lim_{s \to \infty} [sF(s)] = \lim_{t \to 0} f(t).}$$

2) Final Value Theorem:-

$$L\{f(t)\} = F(s)$$

$$\Rightarrow \lim_{t\to\infty} f(t) = \lim_{s\to 0}[sF(s)]$$ Provided the limit exists

Sol

$$L\{f'(t)\} = sF(s) - f(0)$$

$$\int_0^\infty e^{-st} f'(t)\, dt = sF(s) - f(0)$$

$$\lim_{s\to 0}\int_0^\infty e^{-st}\cdot f'(t)\, dt = \lim_{s\to 0}[sF(s) - f(0)]$$

or $$\int_0^\infty \left(\lim_{s\to 0} e^{-st}\right)\cdot f'(t)\, dt = \lim_{s\to 0}[sF(s)] - f(0)$$

$$\int_0^\infty f'(t)\, dt = \lim_{s\to 0}[sF(s)] - f(0)$$

$$\Rightarrow \left[f(t)\right]_0^\infty = \lim_{s\to 0}[sF(s)] - f(0)$$

$$\Rightarrow \lim_{t\to\infty} f(t) - f(0) = \lim_{s\to 0}[sF(s)] - f(0)$$

$$\Rightarrow \boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0}[sF(s)].}$$